The pc concept

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Abstract
This paper is about the reasons of why the parsec is a useful astronomical length unit, although it is of the same order of magnitude of the better known light year. We will describe some concepts related to the definition of this magnitude with the aim of motivate teachers to introduce them since the first approximations of students to the theme of length units and conversion factors between them.

Keywords: Astronomy, parsec, units, parallax.

Resumen
Este artículo trata sobre las razones de por qué el parsec es una unidad astronómica de longitud útil, pese a que es del mismo orden de magnitud que el año luz, unidad que es mucho mejor conocida. Describiremos algunos conceptos relacionados con la definición de parsec con el objetivo de motivar su introducción desde las primeras aproximaciones de los estudiantes al tema de las unidades de la longitud y de los factores de conversión entre ellos.

Palabras clave: Astronomía, parsec, unidades, paralaje.

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I. INTRODUCTION

Several physics text books and courses begin with an introductory chapter dedicated to physical magnitudes, units and conversion factors between them [1, 2, 3, 4]. Sometimes teachers use this theme to mention some units that are related to the basic SI units, such as the electron volt ($1eV=1.602\times10^{-19}$ J) and the atomic mass unit ($1u = 1.660\times10^{-27}$kg). This paper will deal with other example of such units: The **parsec** (symbol **pc**), a unit of length often used in astronomy and cosmology that is equal to about 3.2616 light-years (ly), another length unit (not time unit, as many people belief). People learning for the first time about that, often ask why one uses the former if both units are of the same order of magnitude. Or why one uses a pc if, apparently, a ly unit is easier to understand: it corresponds to the distance in vacuum that the light crosses in a year, i.e. 9.461.000.000.000 Km, while the **parsec** comes from "**par**allax of one arc**sec**ond", so that it is defined as the distance an object has to be from the earth so that its parallax is one arcsecond. Thus the reason why the parsec is very useful to scientists is that the estimation of the distance of a celestial object from the earth must involve the concept of parallax angle. This assertion deserves special attention and its explanation should be presented to students since their very first approximations to these questions. Thus in this paper we will briefly describe what a pc is and how a simple analysis allows the derivation of the relationship or conversion factor between it and 1 ly given above. The way in which this subject can be experimentally treated in schools will be discussed too.

II. PARALLAX

Parallax is what happens when we hold our thumb at arm's length from our face and look at it against the background of our room first with the right eye open and the left eye closed and then with the left eye open and the right eye closed. We will note different backgrounds to our thumb as a result of the slight difference in the relative position of it and each eye. The parallax angle is defined as the half of the angle formed between the right eye, the reference thumb and the left eye, and depends on two things: the distance between both eyes (we will call it the baseline) and the distance to which we locate the finger. If the eyes are quite separated the one of the other, this angle becomes greater; if we moved away the finger, the angle becomes smaller. The word parallax comes from the Greek παράλλαξις (parallaxis), meaning "alteration". Parallax can be exploited to determining the distance of a nearby object: The length of a baseline can be accurately measured and from both ends of it the angle to the nearby object is determined and basic trigonometry is applied to determine the distance to the object, as we will see later.

III. THE PARSEC

The same occurs when we look at a star situated at point D (Fig. 1). Imagine that we look it in one instant from a position E and then we look at the same star six months later, when the Earth is in the opposite side (at E_{+6}) of its orbit around the Sun (at S). The star will appear slightly shifted respecting a background of "fixed" stars (see figure caption for explanation). The distance from the Sun at which this star has to be so that this shift becomes one second of 360 degrees of arc (one arcsecond) is just a parsec.

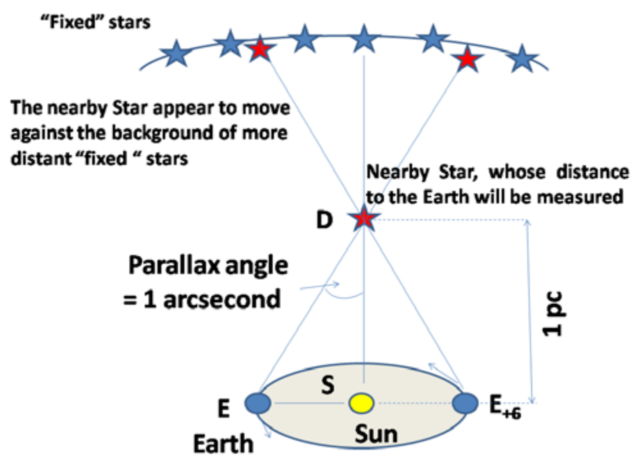

FIGURE 1. Illustrating the definition of pc. The blue colored stars appear as fixed to us because they are located at much greater distances, so that their parallax become so small that it is totally inappreciable to us. In order to understand these concepts it can be of utility this example: when we travel in a car and we watched the near trees, these seem to move with respect to our position. For a same base line (for example a line passing through two reference positions, namely an initial one and an end one of the vehicle trajectory) the more distant trees seem to move slowly, whereas the near one, quickly. This is due to the fact that for the near trees the parallax angle is greater, in such a way that this one changes quickly when the car only moves a small section. The distant trees seem to move less because for them the parallax angle is small.

IV. A VERY SMALL BY MEASURABLE MAGNITUDE

The magnitude of this angular distance is very small if we measure it looking directly with our eyes: 1 second (1") of arch is 1/60 part of a minute of arc (1') which is 1/60 part of a degree. The star next to the Sun is Proxima Centauri (Table I), which is to about 4,3 light years from the Earth or, what is the same, about 1,3 pc. It has a parallax of 0.747" of arc. Most distant stars will undergo an inferior parallax. For example, a star that is 100 pc away from the earth shows a very small parallax of 0,01" of arc. This is similar to being able to observe how a person located at distance of 100 km from us moves a cm to one side. For example, the stars that are kiloparsecs away from us have a parallax of thousandth of second of arc. This is something impossible to appreciate with naked eyes but it is observable and measurable using the great astronomical observatories.

V. CALCULATING THE pc

Now we will make use of trigonometry to calculate a pc from the definition given above. Before we do that, let us define another useful astronomical distance, the so-called astronomical unit (abbreviated as au) that is equal to about 149597871 m, namely the mean distance between the Earth and the Sun over one Earth orbit. In the diagram represented in the Fig. 2 (not to scale), S represents the Sun, and E the Earth at one point in its orbit. Thus the distance ES is just 1 au. If the angle SDE is one arcsecond, then by definition D is a point in space at a distance of one parsec from the Sun, i.e. SD=1pc. By trigonometry, this distance is

$$\mathrm{SD} = 1\mathrm{pc} = \frac{\mathrm{ES}}{\tan(1'')}. \qquad (1)$$

Assuming that 1" is very small we can approximate tangent function at the denominator by its argument becoming:

$$1\mathrm{pc} = \mathrm{SD} \approx \frac{\mathrm{ES}}{1''} = \frac{360\times60\times60}{2\pi}\mathrm{au}, \qquad (2)$$

$$\approx 206264.8\ \mathrm{au} = 206264.48 \times 149597871\mathrm{m},$$

or

$$1\mathrm{pc} \approx 3.085678 \times 10^{16}\mathrm{m}\ . \qquad (3)$$

Thus, taking into account the light year definition we obtain:

$$1\mathrm{pc} \approx 3.261564\ \mathrm{ly}\ . \qquad (4)$$

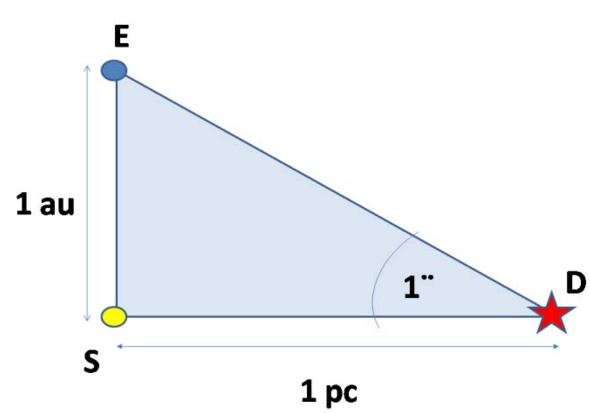

FIGURE 2. Illustrating the definition of pc.

VI. MEASURING THE DISTANCE TO NEARBY STARS

In the same way astronomers can measure the distance to nearby stars (which will appear to move against the background of the more distant "fixed" stars) by taking two images of the night sky six months apart, and applying the basic trigonometry described above. The angle $\sphericalangle$ (SDE) (the angular parallax) of Figures 1 and 2 is measured from these images and then the distance to the nearby star can be calculated according to:

$$\mathrm{SD} = \frac{\mathrm{SE}}{\tan(\sphericalangle(\mathrm{SDE}))}. \quad (5)$$

The distance from the Earth to the Sun, SE, is by definition 1 au. The angle $\sphericalangle$ (SDE) is usually measured in units of arcseconds. As seen above, when this angle is small a valid approximation is tan($\sphericalangle$ (SDE)) ≈ $\sphericalangle$(SDE).

Then, by definition, SD will be expressed in units of pc:

$$\mathrm{SD\ (pc)} = \frac{1\ \mathrm{au}}{\sphericalangle(\mathrm{SDE})\ (\mathrm{arcsec})}. \quad (6)$$

It is worth to notice that the parallax technique is limited to computing the distances to nearby stars. The distance to more distant stars can be measured by other methods, sometimes involving the concepts of star luminosity and period, which should be handled in advanced courses and are behind the scope of this paper.

Table I. Distance in parsecs of some astronomical objects from the earth. A good class room exercise could be to find the equivalent values in ly as well as the corresponding parallax angles. Students should be encourage searching for the distances to the Earth of other stellar objects and about the methods used to measure them.

Object	Distance in pc
Proxima Centauri (nearest known star to our Sun)	1.29
Center of the Milky Way	30×10^3
Virgo Cluster (the nearest large galaxy cluster)	18×10^6
Andromeda Galaxy	0.7×10^6

VII. EXPERIMENTS FOR SCHOOL

On the other hand, for educational purposes waiting six months to make these measurements is not useful and the required equipment is often not available, although good telescopes are obtainable in the market at relative low prices. In 1972 De Jong [5] suggested a simple class room laboratory exercise based in a parallax method described above, where a Polaroid camera (an advanced instrument in that year) was used to take photographs of a lamp from two positions. From the measured parallax and from the well known value of the base line, the distance to the lamp was straightforwardly calculated. De Jong´s experiment can be performed today using a simple digital camera and a cardboard star [6]. This nice exercise can be conducted, for example, on a football field near the school. Several virtual experiments suitable for use in distance education can be also found in Ref. [6].

We hope that this paper can be useful to motivate instructors to teach these questions at schools since the very first years of physics class.

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